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2000 J. Phys.: Condens. Matter 12 4613

(http://iopscience.iop.org/0953-8984/12/21/306)

J. Phys.: Condens. Matter **12** (2000) 4613–4627. Printed in the UK PII: S0953-8984(00)10582-X

The nonlinear effect of transport current on the response of metals to electromagnetic radiation

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Received 4 January 2000, in final form 31 March 2000

Abstract. The nonlinear interaction of a direct current (DC) flowing in a thin metal film with an external low-frequency alternating-current (AC) electromagnetic field is studied theoretically. The nonlinearity is related to the influence of the magnetic field of the DC and the magnetic field of the wave on the form of the electron trajectories. This magnetodynamic mechanism of nonlinearity is typical for pure metals at low temperatures. We find that such an interaction causes sharp kinks in the temporal dependence of the AC electric field of the wave on the surface of a sample. The phenomenon of amplification of the electromagnetic signal on the metal surface is predicted. We also calculate the nonlinear surface impedance and show that it decreases drastically with the increase of the wave amplitude.

1. Introduction

It is already known that metals possess quite peculiar nonlinear electrodynamic properties (see, e.g., references [1, 2]). Usually, in plasmas or semiconductors, a nonlinear response to electromagnetic perturbation is achieved owing to considerable departure of the electron system from equilibrium. In metals, because of the high concentration, electrons are always in near-equilibrium states. Nevertheless, it is fairly standard to observe a nonlinear regime there, which is due to the fact that in metals the sources of nonequilibrium and nonlinearity are different. The former is caused by a weak electric field, while the latter is caused by a strong magnetic field of an electromagnetic wave. The Lorentz force, determined by the magnetic wave component or the magnetic field of the transport current, affects the dynamics of charge carriers. Hence, the conductivity of a metal depends on the configuration of the magnetic field. Such a magnetodynamic mechanism of nonlinearity is typical for pure metals at low temperatures, when the mean free path of conducting electrons is rather large.

Magnetodynamic nonlinearity causes a number of nontrivial electrodynamic phenomena. As an example, one could mention the generation of the *current states* [3, 4] in a sample placed in an external DC magnetic field. According to [3,4], in a plate irradiated by an electromagnetic wave a closed direct current and, as a consequence, an intrinsic DC magnetic moment appear. The magnitude of the magnetic moment depends in a hysteretic manner on the external DC magnetic field. Under current states conditions, a hysteresis-like interaction of radio waves [5] as well as the appearance of electromagnetic dissipative structures [6] is observed. This specific mechanism of nonlinearity results in a decrease of the collisionless damping of helicons [7]. Therefore, spiral waves with large amplitudes can propagate under conditions where the linear electromagnetic excitations vanish [8]. Magnetoplasma shock waves [9] and soliton-like excitations [10] are also predicted for the regime of strong magnetodynamic nonlinearity.

In the present paper, we study a novel manifestation of magnetodynamic nonlinearity, namely, the interaction of an external electromagnetic wave and a strong direct transport current in a thin metal film, which is also displayed in a quite unusual way. The thickness of the sample d is assumed to be much less than the electron mean free path l:

$$d \ll l \tag{1}$$

and electron scattering on the surface of the film is supposed to be diffuse. It is known [11] that in the static case (where the external AC signal is absent), the magnetic field of a current can fundamentally affect the conductivity of a thin metal specimen and, thus, its current–voltage characteristics (CVC). In this situation the value I of the current is rather small, so the typical radius of curvature $R(I)$ of the electron trajectories in a magnetic field is much greater than the film thickness:

$$d \ll R(I) \qquad R(I) = cp_F/eH(I) \propto I^{-1}. \tag{2}$$

Here $-e$ and p_F are the electron charge and Fermi momentum, respectively. In reference [11], it was shown that nonlinear behaviour of the CVC is connected with the antisymmetric spatial distribution of the magnetic field of the direct current over the sample thickness. The magnetic field equals zero at the middle of the film and takes the values H to $-H$ at the opposite boundaries, where

$$H = 2\pi I/cD. \tag{3}$$

In this formula, c denotes the speed of light in vacuum and D is the sample width. The spatially alternating field of the direct current entraps some of the electrons in a potential well. The trajectories of such particles are flat curves winding around the plane where the magnetic field changes sign. The proportion of *trapped* electrons is equal, in order of magnitude, to the typical angle $(d/R)^{1/2} \ll 1$ at which they cross this plane. Taking into account that the trapped carriers do not collide with the film boundaries and interact with the electric field along their whole free path l, one can write the following estimation formula for their conductivity σ_{tr}:

$$\sigma_{tr} \sim \sigma_0 (d/R(I))^{1/2} \propto I^{1/2}. \tag{4}$$

Here σ_0 represents the conductivity of the bulk sample. At the same time, there exist *flying* electrons which do collide with the boundaries of the specimen and, according to reference [12], have conductivity of the order of $\sigma_0(d/l)$. Apparently, in the range of rather strong currents, when the inequality

$$(dR(I))^{1/2} \ll l \tag{5}$$

holds, the conductivity of the film is determined by the group of trapped carriers. As a result, we observe a deviation from Ohm's law: the voltage U is proportional to the square root of the current:

$$U \propto I^{1/2}. \tag{6}$$

For a film with thickness $d = 10^{-3}$ cm, electron mean free path $l = 10^{-1}$ cm, and Fermi momentum $p_F = 10^{-19}$ g cm s^{-1}, the nonlinearity becomes noticeable ($(dR)^{1/2} \sim l$) at values of the magnetic field $H(I)$ of about 1 Oe. The theory developed in reference [11] is in good qualitative agreement with experimental data (see, e.g., reference [13]).

In an external magnetic field $\boldsymbol{h}$, collinear with the magnetic field of the current, the plane of sign alternation for the magnetic field shifts to one of the two boundaries of the film (see figure 1). That, in turn, leads to an appreciable reduction in the number of trapped particles and, therefore, their conductivity. In particular, such a situation would arise under symmetrical irradiation of the film by a large-amplitude low-frequency electromagnetic wave. At low frequencies, the AC magnetic field $\boldsymbol{h}(t)$ of the wave is virtually uniform across the metal (i.e., the wave penetration depth δ is much greater than the sample thickness d). Thus the conductivity of the metal is essentially dependent on time and, therefore, strong nonlinear effects in the sample response to the AC electromagnetic excitation should appear. Although it is of interest from both the theoretical and experimental points of view, this problem has not been investigated yet.

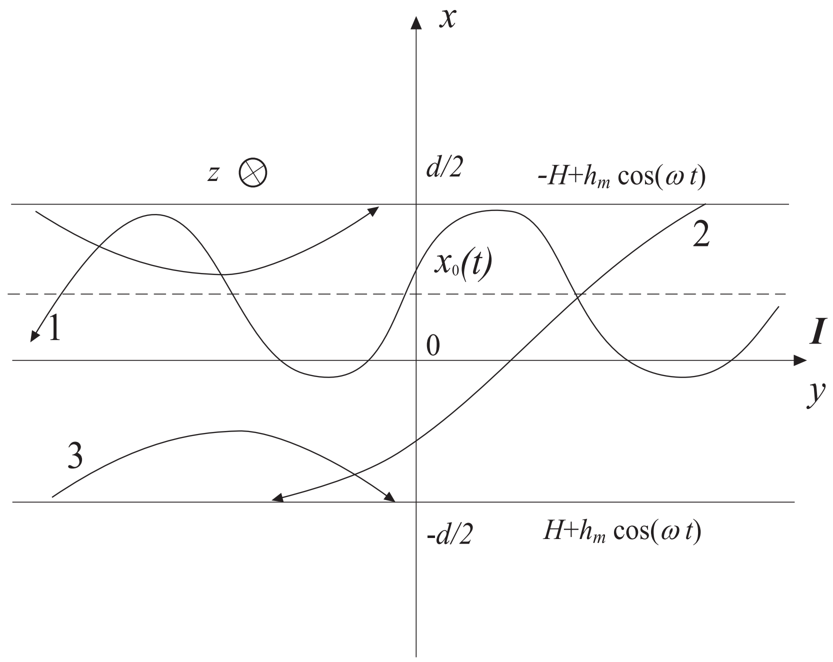

Figure 1. The geometry of the problem. A schematic representation of the trajectories of trapped (1), flying (2), and surface (3) electrons.

In the present paper we study theoretically the temporal dependence of the electric field at the surface of the film, which carries a strong direct current of fixed value I satisfying inequalities (2) and (5). It is shown that with increase of the amplitude h_m of the AC magnetic field, this dependence becomes anharmonic, turning into a series of sharp nonanalytical peaks. The case of sufficiently high amplitudes $h_m > H$, where the total magnetic field in the sample is spatially alternating during some part of the wave period and has a constant sign during the other part, is of particular interest. In such a situation, there exist additional kinks in the temporal dependence of the electric field due to the periodic appearance and disappearance of the group of trapped carriers. The effect of amplification of the electric signal on the film surface is predicted as well. We find that, because of the presence of the strong direct transport current in the sample, the absolute value of the AC electric field of the wave turns out to be $l/(dR)^{1/2} \gg 1$ times greater than the corresponding value in the absence of the direct current. We also calculate the nonlinear surface impedance of the film, which proves to be imaginary in the main approximation in the parameter $d/\delta \ll 1$, and show that it diminishes monotonically $l/(dR)^{1/2} \gg 1$ times as the AC amplitude grows.

2. Problem statement and geometry

Consider a metal film of thickness d with a direct current I flowing along it. The sample is irradiated from both sides by a monochromatic electromagnetic wave with a magnetic component collinear with the magnetic field of the current. The x-axis is oriented perpendicularly to the film boundaries. The plane $x = 0$ corresponds to the middle of the sample (see figure 1). The y-axis is directed along the current direction, and the z-axis is parallel to the vector $\boldsymbol{\mathcal{H}}(x,t)$ of the total magnetic field which is a sum of the magnetic field of the current $\boldsymbol{H}(x,t)$ and the magnetic field of the wave $\boldsymbol{h}(x,t)$:

$$\boldsymbol{\mathcal{H}}(x,t) = \{0, 0, H(x,t) + h(x,t)\}. \tag{7}$$

The film length L (the size along the y-axis) and its width D (the size along the z-axis) are much greater than the sample thickness d. We assume diffuse scattering of the electrons on the film boundaries. Maxwell's equations in the assumed geometry can be written as

$$-\frac{\partial \mathcal{H}(x,t)}{\partial x} = \frac{4\pi}{c} j(x,t) \qquad \frac{\partial E(x,t)}{\partial x} = -\frac{1}{c}\frac{\partial \mathcal{H}(x,t)}{\partial t} \tag{8}$$

where $j(x,t)$ and $E(x,t)$ represent the y-components of the current density and the electric field. The boundary conditions for equations (8) are

$$\mathcal{H}(\pm d/2, t) = h_m \cos\omega t \mp H \tag{9}$$

with H being the absolute value of the magnetic field on the surface of the metal film and h_m denoting the wave amplitude. According to equation (3), the field H is determined by the direct current I. No special relation between the magnitudes H and h_m is assumed.

We focus on the quasistatic situation where the wave frequency ω is much less than the relaxation frequency ν of the charge carriers:

$$\omega \ll \nu. \tag{10}$$

Here we suppose that the AC magnetic field in the sample is quasi-uniform and virtually does not differ from its value on the sample surface, $h(x,t) \simeq h_m \cos\omega t$. In other words, the typical spatial scale $\delta(\omega)$ of variation of the AC magnetic field is much greater than the film thickness d. Furthermore, we assume that the curvature radius $R(x,t)$ of the electron trajectories in the total magnetic field $\mathcal{H}(x,t)$ is also much greater than d:

$$d \ll \delta(\omega) \qquad d \ll R(x,t) \qquad R(x,t) = cp_F/e|\mathcal{H}(x,t)|. \tag{11}$$

3. Electron dynamics, current density, and the CVC of the film

Let us consider the electron dynamics in the nonuniform AC magnetic field $\mathcal{H}(x,t)$. We shall assume the following gauge for the vector potential:

$$\boldsymbol{A}(x,t) = \{0, A(x,t), 0\} \qquad A(x,t) = \int^{x} \mathrm{d}x'\, \mathcal{H}(x',t). \tag{12}$$

It is convenient to choose the lower limit of integration in equation (12) depending on whether or not there exists a plane $x = x_0(t)$ of sign alternation of the magnetic field $\mathcal{H}(x,t)$ at the given moment. This plane exists during the time intervals when $h_m|\cos\omega t| < H$ because the values $h_m \cos\omega t - H$ and $h_m \cos\omega t + H$ of the total magnetic field at the film boundaries have opposite signs (see equation (9)). In this case, one should take $x_0(t)$ as the lower limit in integral (12). Then the vector potential $A(x,t)$ is negative. It reaches its maximum value (which equals zero) at the point $x = x_0(t)$. Within other time intervals, when the inequality $h_m|\cos\omega t| > H$ holds, the magnetic field $\mathcal{H}(x,t)$ is of constant sign. In such a situation, one should choose

$\text{sgn}(\cos\omega t)d/2$ ($\text{sgn}(x)$ is the signum function) as the lower limit of the integration. In this case, the vector potential, also being negative, vanishes at one of the boundaries of the film.

The integrals of motion of an electron in the field $\mathcal{H}(x, t)$ are the total energy (it is equal to the Fermi energy) and the canonical momenta $p_z = mv_z$ and $p_y = mv_y - eA(x, t)/c$ (m is the electron mass). The electron trajectory in the plane perpendicular to the direction of the magnetic field is determined by the velocities $v_x(x, t)$ and $v_y(x, t)$. For the case of a Fermi sphere of radius $p_F = mv$ we obtain

$$|v_x(x,t)| = (v_\perp^2 - v_y^2)^{1/2}$$
$$v_\perp = (v^2 - v_z^2)^{1/2} \tag{13}$$
$$v_y(x,t) = (p_y + eA(x,t)/c)/m.$$

Classically admissible regions of the electron motion along the x-axis are determined by the inequalities

$$-p_y - mv_\perp \leqslant eA(x,t)/c \leqslant -p_y + mv_\perp. \tag{14}$$

These inequalities guarantee the positivity of the radicand in equation (13) for $|v_x(x, t)|$.

The regions of the electron motion in the phase plane (x, p_y) are shown schematically in figure 2 for two cases: when there exists a plane $x = x_0(t)$ of sign alternation of the magnetic field $\mathcal{H}(x, t)$ (figure 2(a)) and when such a plane is absent (figure 2(b)). For definiteness, we have chosen the moment of time when the magnetic field of the wave is positive ($\cos\omega t > 0$). The upper border of the phase plane is described by the curve $p_y = mv_\perp - eA(x, t)/c$ and the lower one is given by $p_y = -mv_\perp - eA(x, t)/c$. The electrons are naturally divided into three groups depending on the sign and value of the integral of motion p_y. Below, we give inequalities determining the regions of their existence at an arbitrary moment of time.

(a) Flying electrons:

$$p_y^- \equiv -mv_\perp - eA[-\text{sgn}(\cos\omega t)d/2, t]/c \leqslant p_y \leqslant mv_\perp \qquad |x| \leqslant d/2. \tag{15}$$

These particles collide with the both boundaries of the film. Their trajectories do not twist significantly because $d \ll R(x, t)$. Flying electrons exist at every moment of time

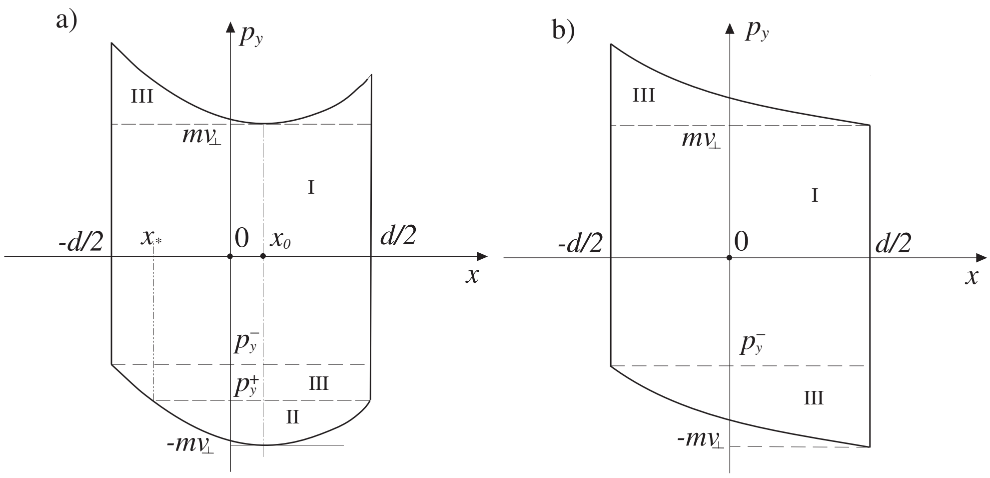

Figure 2. The phase space (p_y, x). Regions of existence of flying (I), trapped (II), and surface (III) particles in a spatially sign-alternating (a) and a constant-sign (b) total magnetic field.

irrespective of the presence of the plane $x = x_0(t)$ (i.e. irrespective of the relation between $h_m \cos\omega t$ and H).

(b) Trapped electrons: they appear during the periods of time when $h_m|\cos\omega t| < H$ and the total magnetic field $\mathcal{H}(x, t)$ within the sample passes through zero. Their states are bounded by the region (see figure 2(a))

$$\begin{aligned} -mv_\perp \leqslant p_y \leqslant p_y^+ \equiv -mv_\perp - eA[\mathrm{sgn}(\cos\omega t)d/2, t]/c \\ x_*(t)\,\mathrm{sgn}(\cos\omega t) < x\,\mathrm{sgn}(\cos\omega t) < d/2. \end{aligned} \tag{16}$$

Here $x_*(t)$ represents the breakpoint of the trapped electron most distant from the film boundary. One can find it from the equation

$$A(x_*, t) = A[\mathrm{sgn}(\cos\omega t)d/2, t]. \tag{17}$$

According to equation (16), this electron group occupies the region $x_*(t) < x < d/2$ when $\cos\omega t > 0$ and the region $-d/2 < x < x_*(t)$ if $\cos\omega t < 0$. The trajectories of trapped particles are almost flat oscillating curves due to the periodic motion of the particles along the x-direction and the uniform motion along the y- and z-axes. The temporal period of the oscillations with respect to the plane $x = x_0$ equals $2T$, where

$$T = \int_{x_1(t)}^{x_2(t)} \frac{\mathrm{d}x}{|v_x(x, t)|}. \tag{18}$$

The breakpoints $x_1(t)$ and $x_2(t)$ ($x_1(t) < x_0(t) < x_2(t)$) are the roots of the equation

$$eA(x_{1,2}, t)/c = -mv_\perp - p_y. \tag{19}$$

(c) Surface electrons: these particles collide only with one of the boundaries of the film. In our case of diffuse scattering of the electrons on the surface, their influence on the nonlinear conductivity of the metal is negligible [11]. Thus, we do not take them into account hereafter.

The current density of the flying and trapped particles can be deduced by means of solving the Boltzmann kinetic equation. One should linearize the kinetic equation with respect to the electric field $E(x, t)$, which can be represented as a sum

$$\begin{aligned} E(x, t) = E_0 + \mathcal{E}(x, t) \\ \mathcal{E}(x, t) = -\frac{1}{c}\left(\frac{\partial A(x, t)}{\partial t} - \frac{\partial \bar{A}(t)}{\partial t}\right). \end{aligned} \tag{20}$$

Here the first term, E_0, is a potential (uniform) component and $\mathcal{E}(x, t)$ is a rotational (non-uniform) field of the wave. Spatial averaging of the latter over the x-axis direction gives zero. The value $\bar{A}(t)$ represents a spatially averaged magnitude of the vector potential:

$$\bar{A}(t) = \frac{1}{d}\int_{-d/2}^{d/2} A(x', t)\,\mathrm{d}x'. \tag{21}$$

The magnetodynamic nonlinearity is accounted for in the kinetic equation by means of terms which contain the total magnetic field $\mathcal{H}(x, t) = H(x, t) + h(x, t)$ entering the Lorentz force. We calculate the current density in the main approximation with respect to the small parameter $d/\delta(\omega)$ (see equation (11)). In this approximation, as was mentioned above, the AC magnetic field $h(x, t)$ becomes spatially uniform and is equal to its boundary value, $h(x, t) = h_m \cos\omega t$. The electric field is also independent of the x-coordinate and coincides with the value $E_0(t)$. For the case of uniform electric and external magnetic fields, the current density was obtained in reference [11]. If conditions (2) and (5) hold, the following asymptotic behaviour for the current density of the flying and trapped electrons is valid:

$$j_{fl}(t) = \sigma_{fl}(t)E_0(t)$$
$$\sigma_{fl}(t) = \frac{3}{8}\sigma_0\frac{d}{l}\ln\frac{R_+(t)}{d} \qquad R_\pm(t) = cp_F/e|h_m|\cos\omega t| \pm H| \tag{22}$$
$$j_{tr}(x,t) = \sigma_{tr}(x,t)E_0(t)$$
$$\sigma_{tr}(x,t) = \frac{36\pi^{1/2}}{5\Gamma^2(1/4)}\sigma_0\left\{\frac{e}{cp_F}\left[A(x,t) - A(\mathrm{sgn}(\cos\omega t)d/2,t)\right]\right\}^{1/2} \tag{23}$$
$$x_*(t)\,\mathrm{sgn}(\cos\omega t) < x\,\mathrm{sgn}(\cos\omega t) < d/2.$$

In the limit $\omega \to 0$, equations (22) and (23) transform into the corresponding formulae of reference [11].

Let us substitute asymptotic expressions (22) and (23) for the current density in the first of Maxwell's equations (8) and introduce a dimensionless coordinate and vector potential:

$$\xi = 2x\,\mathrm{sgn}(\cos\omega t)/d \qquad a(\xi,t) = A(x,t)/A(\mathrm{sgn}(\cos\omega t)d/2,t). \tag{24}$$

The equation for the quantity $a(\xi,t)$ has the form

$$\frac{\partial^2 a(\xi,t)}{\partial\xi^2} = u\begin{cases} r[1-a(\xi,t)]^{1/2}+1 & \xi_*(t) \leqslant \xi \leqslant 1 \\ 1 & -1 \leqslant \xi \leqslant \xi_*(t) \end{cases} \tag{25}$$

$$\xi_*(t) = 2x_*(t)\,\mathrm{sgn}(\cos\omega t)/d. \tag{26}$$

The dimensionless coordinate $\xi_*(t)$ delimits the region of existence of the trapped particles and, according to equations (17) and (24), satisfies the equation, $a(\xi_*,t) = 1$. The parameter r represents the ratio of the maximum magnitude of the conductivity of the trapped electrons to the conductivity of the flying particles:

$$r = \frac{\sigma_{tr}(x_0)}{\sigma_{fl}} = \frac{96\pi^{1/2}}{5\Gamma^2(1/4)}\frac{l}{d}\left[\frac{e}{cp_F}|A(\mathrm{sgn}(\cos\omega t)d/2,t)|\right]^{1/2}\ln^{-1}(R_+/d). \tag{27}$$

The dimensionless quantity, u, is related to the voltage $U = E_0L$ on the sample:

$$u = \frac{U}{cL|A(\mathrm{sgn}(\cos\omega t)d/2,t)|/\pi\sigma_{fl}d^2}. \tag{28}$$

Equation (25) should be solved together with the boundary conditions

$$\frac{\partial a(1,t)}{\partial\xi} = \frac{d}{2}\frac{h_m|\cos\omega t| - H}{A(\mathrm{sgn}(\cos\omega t)d/2,t)}$$
$$\frac{\partial a(-1,t)}{\partial\xi} = \frac{d}{2}\frac{h_m|\cos\omega t| + H}{A(\mathrm{sgn}(\cos\omega t)d/2,t)} \qquad a(1,t) = 1. \tag{29}$$

The first two of these expressions are dimensionless boundary conditions (9), and the third one is a consequence of the normalization (24) of the vector potential.

Within the interval $\xi_*(t) \leqslant \xi \leqslant 1$, the solution of equation (25) is symmetrical with respect to the point $\xi_0(t) = (1+\xi_*(t))/2$, where the dimensionless vector potential reaches its minimum value (which equals zero, $a(\xi_0,t) = \partial a(\xi_0,t)/\partial\xi = 0$). This solution is described by the formula

$$|\xi - \xi_0(t)| = (3/4ru)^{1/2}\int_0^{a(\xi,t)} \mathrm{d}\zeta\,\left[1-(1-\zeta)^{3/2} + 3\zeta/2r\right]^{-1/2}. \tag{30}$$

One cannot obtain the field distribution and the current density within the region of existence of the trapped electrons in an explicit form. However, by means of equation (30), it is possible to calculate the average magnitude of the conductivity of the trapped carriers (23) within the interval (16)

$$\frac{\bar{\sigma}_{tr}}{\sigma_{fl}} = r\int_0^1 \mathrm{d}\zeta\ (1-\zeta)^{1/2}[1-(1-\zeta)^{3/2}+3\zeta/2r]^{-1/2} \times \left(\int_0^1 \mathrm{d}\zeta\ [1-(1-\zeta)^{3/2}+3\zeta/2r]^{-1/2}\right)^{-1}. \tag{31}$$

The bar above σ_{tr} denotes spatial averaging. In the remaining region of the sample ($-1 \leqslant \xi \leqslant \xi_*(t)$), there exist only flying electrons, and the solution of equation (25) is given by the following formula:

$$a(\xi, t) = 1 - (2u)^{1/2}(1+2r/3)^{1/2}(\xi - \xi_*(t)) + u(\xi - \xi_*(t))^2/2. \tag{32}$$

Expressions (30) and (32) and their derivatives are sewn together at the point $\xi = \xi_*(t)$. The solution given by equations (30) and (32) contains three parameters, ξ_0, u, and r, which should be found from boundary conditions (29). It is important that the value $A(\mathrm{sgn}(\cos\omega t)d/2, t)$ of the vector potential appearing in equation (29) is not an independent parameter due to its relation to r via formula (27).

Adding the first two boundary conditions in equation (29) term by term and using equations (30) and (32), and (28), we find the following expression for the drift of the plane $x = x_0$:

$$\xi_0 = 2x_0\,\mathrm{sgn}(\cos\omega t)/d = \frac{Lh_m|\cos\omega t|}{2\pi U\sigma_{fl}d} \qquad h_m|\cos\omega t| \leqslant H. \tag{33}$$

In order to determine the value of u (i.e. the voltage U), let us integrate the left- and right-hand sides of equation (25) from -1 to 1 taking into account the boundary conditions for the derivative $\partial a(\xi, t)/\partial\xi$ in (29). The integral of the function $[1 - a(\xi, t)]^{1/2}$ appearing on the right-hand side can be reduced to the product $2(1-\xi_0)\bar{\sigma}_{tr}/r\sigma_{fl}$ with the use of the condition $a(1, t) = 1$. Taking this into consideration as well as formulae (28) and (33) for the quantities u and ξ_0, we have after some simple transformations

$$U = \frac{cL}{2\pi d\sigma_{fl}(t)}\frac{H(I) + (\bar{\sigma}_{tr}/\sigma_{fl})h_m|\cos\omega t|}{1 + \bar{\sigma}_{tr}/\sigma_{fl}} \qquad h_m|\cos\omega t| \leqslant H. \tag{34}$$

According to equation (31), the ratio of conductivities, $\bar{\sigma}_{tr}/\sigma_{fl}$, depends on the parameter r. Using expression (28) for u, relation (27) between $A(\mathrm{sgn}(\cos\omega t)d/2, t)$ and r, and solution (30), we obtain from the first boundary condition in equation (29) the algebraic equation for r:

$$r^2(1+2r/3) = \left(\frac{H - h_m|\cos\omega t|}{\tilde{H}}\right)^2 \frac{\tilde{U}}{U\ln^3(R_+/d)} \qquad h_m|\cos\omega t| \leqslant H. \tag{35}$$

Here we have introduced the following notation:

$$\tilde{H} = \frac{25\Gamma^4(5/4)}{9\pi}\frac{cp_F d}{el^2} \qquad \tilde{U} = \frac{4clL\tilde{H}}{3\pi\sigma_0 d^2}. \tag{36}$$

The parameters $\tilde{H}$ and $\tilde{U}$ represent those magnitudes of the magnetic field and voltage for which the typical length $(Rd)^{1/2}$ of the electron trajectory arc is of the order of the mean free path l.

Expressions (31), (34), and (35) define, in an implicit form, the dependence of the voltage U on the current I for the case where $h_m|\cos\omega t| \leqslant H$. Under these conditions there exists a plane of alternation of sign of the total magnetic field within the sample. If the opposite inequality, $h_m|\cos\omega t| \geqslant H$, is valid, the trapped electrons are absent ($r = 0$, $\xi_* = 1$, $\sigma_{tr} = 0$) and the CVC is described by the formula

$$U = \frac{cLH(I)}{2\pi d\sigma_{fl}(t)} \qquad h_m|\cos\omega t| \geqslant H. \tag{37}$$

As one can see from formula (34), the voltage on the sample displays nonanalytical behaviour versus time: the dependence $U(t)$ has kinks at the moments when the AC magnetic field $h_m \cos\omega t$ vanishes. This is an essentially nonlinear effect caused by the contribution of a large group of trapped electrons to the electric current. The temporal dependence of the voltage (34) for the case where the wave amplitude is not too large ($h_m < H$) and there exist trapped carriers throughout the whole period $2\pi/\omega$ is shown in figure 3(a); figure 3(b) represents the dependence $U(t)$ for the opposite case, $h_m > H$, in which during some part of the wave period (for $h_m|\cos\omega t| \geqslant H$), the conductivity is caused by the flying particles only.

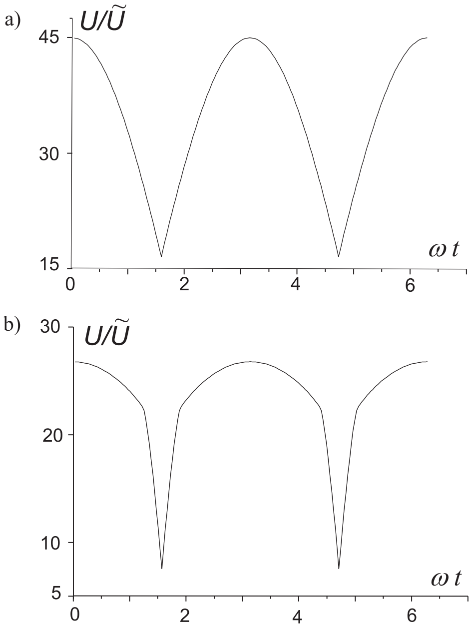

Figure 3. The time dependence of the voltage U at relatively small ((a) $h_m < H$) and large ((b) $h_m > H$) AC amplitudes.

4. Nonanalytical temporal dependence of the electric field

The knowledge of the vector potential $A(x, t)$ allows one to calculate the rotational electric field $\mathcal{E}(x, t)$ as a correction to $E_0(t)$ (see equation (20)). We are interested in the difference $\Delta\mathcal{E}(t) = \mathcal{E}(d/2, t) - \mathcal{E}(-d/2, t)$. This value is proportional to the rate of alteration of the magnetic flux through the cross-sectional plane, which is perpendicular to the direction of the vector of the total field $\mathcal{H}(x, t)$, and thus can be measured in experiment.

From equations (30) and (32), it follows that the difference $a(1, t) - a(-1, t)$ is connected with the derivatives $\partial a(1, t)/\partial\xi$ and $\partial a(-1, t)/\partial\xi$ by the relations

$$a(1, t) - a(-1, t) = -\xi_0(t)\left[\frac{\partial a(1, t)}{\partial\xi} - \frac{\partial a(-1, t)}{\partial\xi}\right] \qquad h_m|\cos\omega t| \leqslant H \tag{38}$$

$$a(1,t) - a(-1,t) = \frac{\partial a(1,t)}{\partial \xi} + \frac{\partial a(-1,t)}{\partial \xi} \qquad h_m|\cos\omega t| \geqslant H. \tag{39}$$

Let us now turn to the dimensional variables in equations (38) and (39) using boundary conditions (29) and relation (27) between the values of $A(\mathrm{sgn}(\cos\omega t)d/2, t)$ and r. After that one can obtain the following expression for the magnitudes of the vector potential at the film boundaries:

$$\begin{aligned} &A(\mathrm{sgn}(\cos\omega t)d/2, t) = -\tilde{H} d \ln^2(R_+/d) r^2/4 \\ &A(-\mathrm{sgn}(\cos\omega t)d/2, t) = -\tilde{H} d \ln^2(R_+/d) r^2/4 - 2H|x_0(t)| \end{aligned} \tag{40}$$

for

$$h_m|\cos\omega t| \leqslant H$$

and

$$\begin{aligned} &A(\mathrm{sgn}(\cos\omega t)d/2, t) = 0 \\ &A(-\mathrm{sgn}(\cos\omega t)d/2, t) = -d h_m|\cos\omega t| \end{aligned} \tag{41}$$

for

$$h_m|\cos\omega t| \geqslant H.$$

Formulae (40) and (41) are sewn together the time moment when $h_m|\cos\omega t| = H$. The parameter r in equation (27) vanishes, and the plane $x = x_0(t)$ coincides with one of the boundaries of the sample, $|x_0(t)| = d/2$. From relations (40) and (20), by means of formula (33) for $\xi_0(t)$, we derive the expression for the difference $\Delta\mathcal{E}(t)$ of magnitudes of the electric field at the film boundaries:

$$\Delta\mathcal{E}(t) = -\frac{2H}{c}\frac{\partial x_0(t)}{\partial t} = -\frac{H(I)Lh_m}{2\pi}\frac{\partial}{\partial t}\left[\frac{\cos\omega t}{\sigma_{fl}(t)U(t)}\right] \qquad h_m|\cos\omega t| \leqslant H. \tag{42}$$

If the inequality $h_m \leqslant H$ holds, the previous relation is valid throughout the whole period of the wave. However, in the case where $h_m > H$, there exists a time interval when the plane $x = x_0(t)$ of alternation of sign of the total magnetic field is absent. If such a situation arises, one should use formula (41) in order to obtain the dependence $\Delta\mathcal{E}(t)$. Finally we come to the result below:

$$\Delta\mathcal{E}(t) = \Delta\mathcal{E}_L \sin\omega t \qquad \Delta\mathcal{E}_L = d h_m \omega/c \qquad h_m|\cos\omega t| \geqslant H. \tag{43}$$

From this, it follows that the difference $\Delta\mathcal{E}(t)$ is a harmonic function of time, i.e. the response of the film to the external electromagnetic excitation turns out to be linear if there are no trapped electrons. It is obvious that formula (43) also describes the dependence $\Delta\mathcal{E}(t)$ at small magnitudes of the current I ($H \ll \tilde{H}$), when the contribution of trapped particles to the conductivity is negligible throughout the whole period of the wave. Then the value $\Delta\mathcal{E}_L$ represents the amplitude of the linear response.

The dependence $\Delta\mathcal{E}(t)$ is shown in figure 4 for a wide range of the AC amplitudes h_m and for large magnitudes of the DC magnetic field H of the current I, when the inequality $H \gg \tilde{H}$ (or inequality (5)) is valid. It is obvious that the ratio of the amplitude $\Delta\mathcal{E}_m$ to its linear value $\Delta\mathcal{E}_L$ does not depend on h_m. From relations (42), (34), and (35) at $\cos\omega t = 0$, we find the expression for $\Delta\mathcal{E}_m$:

$$\frac{\Delta\mathcal{E}_m}{\Delta\mathcal{E}_L} = 0.83\left(\frac{H}{\tilde{H}}\right)^{1/2}\frac{1}{\ln(R/d)} \qquad \left(\frac{H}{\tilde{H}}\right)^{1/2} \sim \left.\frac{\sigma_{tr}}{\sigma_{fl}}\right|_{\cos\omega t=0} \sim \frac{l}{(Rd)^{1/2}} \gg 1. \tag{44}$$

The ratio $\Delta\mathcal{E}_m/\Delta\mathcal{E}_L$ is determined by the magnitude of the DC magnetic field H and can be much greater than unity. In other words, there exists an effect of amplification of the electric

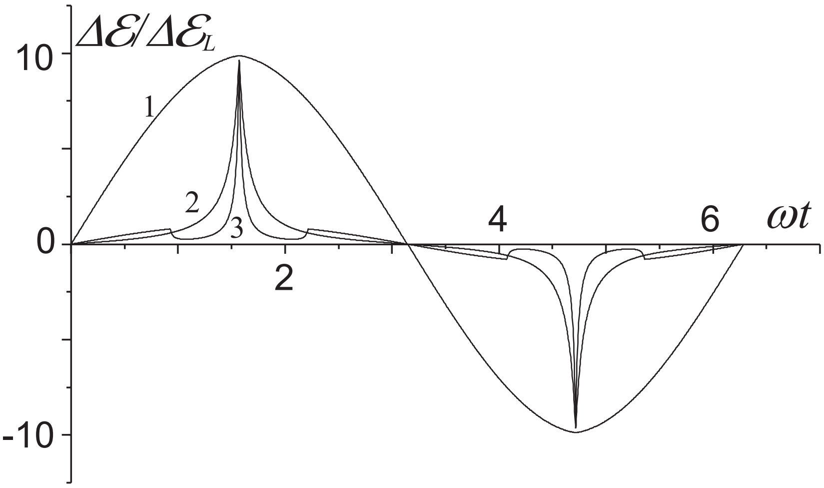

Figure 4. The dependence $\Delta\mathcal{E}(t)$ for $H = 300\tilde{H}$ and various amplitudes of the AC signal: $h_m = 1\tilde{H}$ (1), $h_m = 200\tilde{H}$ (2), $h_m = 500\tilde{H}$ (3). The ratio of the mean free path l to the film thickness d equals 30.

signal at the film surface. For small AC amplitudes (curve 1, $h_m = H/300$), the signal turns out to be quasi-harmonic. However, with the increase of h_m the dependence $\Delta\mathcal{E}(t)$ shows kinks. Curve 2 has kinks at the points of extrema, i.e. at the time moments when the AC magnetic field $h_m \cos \omega t$ vanishes. These singularities are related to the nonanalytical behaviour of the CVC of the film (see equation (34) and figure 3). Curve 3 corresponds to the case where $h_m = 5H/3$, in which the trapped electrons are absent during a part of the wave period. In such a situation, the dependence $\Delta\mathcal{E}(t)$ contains additional kinks arising at the moments of appearance and disappearance of the plane $x = x_0(t)$ of sign alternation of the total magnetic field. They are located symmetrically with respect to the points of extrema as shown by curve 3. By means of formulae (42), (43), (34), and (35), we find the right and left derivatives of the function $\Delta\mathcal{E}(t)$ at the point $t_0 = (1/\omega) \arccos(H/h_m)$ of the first kink:

$$\frac{\partial}{\partial t} \frac{\Delta\mathcal{E}(t)}{\Delta\mathcal{E}_L}\bigg|_{t=t_0-0} = \frac{\omega H}{h_m} \tag{45}$$

$$\frac{\partial}{\partial t} \frac{\Delta\mathcal{E}(t)}{\Delta\mathcal{E}_L}\bigg|_{t=t_0+0} = \frac{\omega H}{h_m}\left[1 - \frac{\pi}{2\ln(R_+/d)}\left(\frac{H}{\tilde{H}}\right)^{1/2}\left(\frac{h_m^2}{H^2} - 1\right)\right]. \tag{46}$$

According to equation (46), the right derivative is negative and has large absolute value even at $[(h_m/H)^2 - 1] \geqslant 1$.

5. Surface impedance of the film

Let us analyse the dependence of the surface impedance at the film boundary $x = d/2$ on the AC amplitude h_m under conditions of interaction of the transport current and the electromagnetic wave. The impedance is proportional to the ratio of the first Fourier harmonics of the electric ($\mathcal{E}_\omega$) and magnetic (h_ω) fields at the surface of the sample:

$$Z = \frac{4\pi}{c}\frac{\mathcal{E}_\omega}{h_\omega} = \frac{8\pi}{c}\frac{\mathcal{E}_\omega}{h_m} \tag{47}$$

where

$$\mathcal{E}_\omega = -\frac{\omega}{2\pi c}\int_0^{2\pi/\omega}\left(\frac{\partial A(d/2,t)}{\partial t} - \frac{\partial \bar{A}}{\partial t}\right)\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}t = \frac{\mathrm{i}\omega^2}{2\pi c}\int_0^{2\pi/\omega}\big(A(d/2,t) - \bar{A}(t)\big)\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}t. \tag{48}$$

Taking into account equations (33) and (40), we deduce the boundary value of the vector potential for the periods of time given by the inequality $h_m|\cos\omega t| \leqslant H$:

$$A(d/2,t) = \begin{cases} -\tilde{H}d\ln^2(R_+/d)r^2/4 & \cos\omega t > 0 \\ -\tilde{H}\ln^2(R_+/d)r^2/4 + cHLh_m\cos\omega t/2\pi U(t)\sigma_{fl}(t) & \cos\omega t < 0. \end{cases} \tag{49}$$

In the case where $h_m|\cos\omega t| \geqslant H$, the following expression is valid (see equation (41)):

$$A(d/2,t) = \begin{cases} 0 & \cos\omega t > 0 \\ dh_m\cos\omega t & \cos\omega t < 0. \end{cases} \tag{50}$$

Let us calculate the mean value of the vector potential $\bar{A}(t)$ for $h_m|\cos\omega t| \leqslant H$, when there exists a plane of alternation of sign of the field. According to equations (30) and (32), we have

$$\begin{aligned}\frac{\bar{A}(t)}{A(\mathrm{sgn}(\cos\omega t)d/2,t)} &= \frac{1}{2}\int_{-1}^{1} a(\xi,t)\,\mathrm{d}\xi \\ &= \xi_0(t) + (2u(t))^{1/2}(1 + r(t)/3)^{1/2}\xi_0^2(t) \\ &\quad + (2/3)u(t)\xi_0^3 + \left(\frac{3}{4r(t)u(t)}\right)^{1/2}\int_0^1 \frac{\zeta\,\mathrm{d}\zeta}{\sqrt{1-(1-\zeta)^{3/2} + 3\zeta/2r(t)}}.\end{aligned} \tag{51}$$

In the case where $h_m|\cos\omega t| \geqslant H$, one should use solution (32) with $r = 0$, $\xi_* = 1$ in order to find $\bar{A}(t)$. Proceeding to dimensional variables and using equations (28) and (37), one can easily obtain

$$\bar{A}(t) = -\frac{dh_m|\cos\omega t|}{2} - \frac{1}{6}Hd. \tag{52}$$

We draw the reader's attention to the fact that the mean value of the vector potential depends on time only via the term $|\cos\omega t|$, $\bar{A}(t) = \bar{A}(|\cos\omega t|)$. This follows from formulae (35), (28), and (33) for the values r, u, and ξ_0 as well as from relation (27) between $A(\mathrm{sgn}(\cos\omega t)d/2,t)$ and r.

We start the calculation of the surface impedance with the case of relatively small amplitudes $h_m < H$, when the group of trapped electrons exists throughout the whole period of the wave. Let us substitute expressions (49) and (51) into equation (47). Then, the integrals containing $\bar{A}(t)$ and $-\tilde{H}d\ln^2(R_+/d)r^2/4$ vanish since these functions depend on $|\cos\omega t|$ only. By means of formula (34) for the voltage U, the remaining integral can be transformed into the form

$$Z = \frac{8\mathrm{i}d\omega}{c^2}\int_0^{\pi/2}\frac{1 + \bar{\sigma}_{tr}(\tau)/\sigma_{fl}(\tau)}{1 + (\bar{\sigma}_{tr}(\tau)/\sigma_{fl}(\tau))(h_m/H)\cos\tau}\cos^2\tau\,\mathrm{d}\tau \qquad h_m \leqslant H. \tag{53}$$

For the case of large amplitudes $h_m > H$, one should calculate the impedance using formulae (49), (50), (51), and (52). It represents a sum of two terms:

$$Z = \frac{8\mathrm{i}d\omega}{c^2}\left[\int_{\pi/2-\arcsin H/h_m}^{\pi/2}\frac{1 + \bar{\sigma}_{tr}(\tau)/\sigma_{fl}}{1 + (\bar{\sigma}_{tr}(\tau)/\sigma_{fl}(\tau))(h_m/H)\cos\tau}\cos^2\tau\,\mathrm{d}\tau + \int_0^{\pi/2-\arcsin H/h_m}\cos^2\tau\,\mathrm{d}\tau\right] \qquad \text{at } h_m > H. \tag{54}$$

The first term corresponds to the temporal interval when the trapped electrons exist in the sample, and the second one is related to the interval when these particles are absent.

Let us calculate the asymptotics of the surface impedance for the case of rather large amplitudes $h_m \gg H$. For this purpose, we rewrite integral (54) in another form:

$$Z = \frac{8\mathrm{i}d\omega}{c^2}\left[\int_0^{\pi/2} \cos^2\tau\,\mathrm{d}\tau + \int_{\pi/2-\arcsin H/h_m}^{\pi/2}\left(\frac{1+\bar{\sigma}_{tr}(\tau)/\sigma_{fl}}{1+(\bar{\sigma}_{tr}(\tau)/\sigma_{fl}(\tau))(h_m/H)\cos\tau} - 1\right)\cos^2\tau\,\mathrm{d}\tau\right]. \tag{55}$$

In the second integral, we substitute in the variable of integration $(h_m\cos\tau)/H = \eta$ and expand the integrand in a power series in the ratio H/h_m. Then one finds

$$\frac{Z}{Z_L} = 1 + \frac{4}{\pi}(H/h_m)^3\int_0^1\left[\frac{1+\bar{\sigma}_{tr}(\pi/2)/\sigma_{fl}(\pi/2)}{1+\bar{\sigma}_{tr}(\pi/2)/\sigma_{fl}(\pi/2)\eta} - 1\right]\eta^2\,\mathrm{d}\eta \tag{56}$$

where

$$Z_L = \mathrm{i}\frac{2\pi}{c^2}\omega d \tag{57}$$

is the same as the value of the impedance in the absence of the direct transport current. The conductivities $\bar{\sigma}_{tr}(\pi/2)$, and $\sigma_{fl}(\pi/2)$ are taken at the moment of time when the AC magnetic field $h_m\cos\omega t$ becomes zero. Therefore, their ratio is much greater than unity due to inequality (44). Taking into account condition (44), we calculate integral (56) and obtain the following asymptotic behaviour for the impedance:

$$\frac{Z}{Z_L} = 1 + \frac{2}{3\pi}\left(\frac{H}{h_m}\right)^3 \qquad H \ll h_m. \tag{58}$$

Now we consider the case of the extremely small amplitudes described by the inequality $h_m \ll H\sigma_{fl}(\pi/2)/\sigma_{tr}(\pi/2) \sim (H\tilde{H})^{1/2}$. The integrand in equation (53) can be presented as a power series in $h_m/(H\tilde{H})^{1/2}$. As a result the asymptotic behaviour takes the form

$$\frac{Z}{Z_L} = \frac{4}{\pi}\frac{\bar{\sigma}_{tr}(\pi/2)}{\sigma_{fl}(\pi/2)}\int_0^{\pi/2}\left[1 - \frac{\sigma_{tr}(\pi/2)}{\sigma_{fl}(\pi/2)}\frac{h_m}{H}\cos\tau\right]\cos^2\tau\,\mathrm{d}\tau = \frac{\sigma_{tr}(\pi/2)}{\sigma_{fl}(\pi/2)}\left(1 - \frac{8}{3\pi}\frac{\sigma_{tr}(\pi/2)}{\sigma_{fl}(\pi/2)}\frac{h_m}{H}\right) \qquad \text{at } h_m \ll (H\tilde{H})^{1/2}. \tag{59}$$

We notice that impedance (59) is $\sigma_{tr}(\pi/2)\sigma_{fl}(\pi/2) \gg 1$ times greater than that in the absence of the direct current. This is a direct consequence of the effect of the amplification of the electric signal at the film boundary which was treated in the previous section (see equation (44)). The presence of a strong direct current in the sample also causes linear behaviour of the impedance in the region of small amplitudes. As shown in figure 5, the impedance decreases monotonically within the region between asymptotics (59) and (58).

6. Conclusions

The nonlinear interaction of electromagnetic waves with a strong direct transport current in a thin metal film leads to unusual effects due to the specific—typical only for metals—*magnetodynamic* nonlinearity mechanism. These effects have been studied by analysing the nonlinear response of the film, which carries the direct current, irradiated bilaterally by an electromagnetic wave. The interaction of the wave with the current results in the nonanalytical behaviour of the AC electric field versus time on the sample surface which is characterized by the appearance

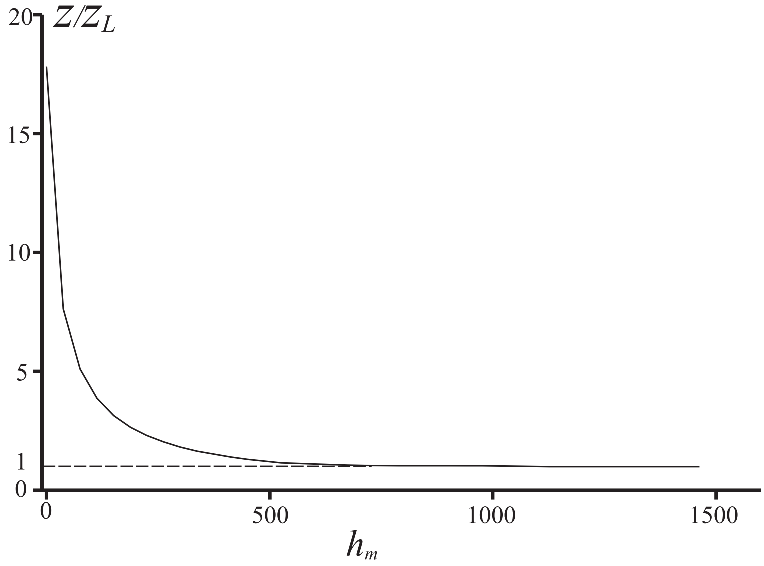

Figure 5. The surface impedance Z (in units of the impedance Z_L in the absence of the direct current) versus the dimensionless amplitude $h_m/\tilde{H}$ of the AC signal at $H = 500\tilde{H}$.

of sharp kinks. The increase of the current is accompanied by a rise of the amplitude of the electric field oscillations at the surface of the sample. This, in turn, causes the growth of the surface impedance of the conductor.

The results obtained in this paper are valid under certain applicability conditions. Firstly, the AC electric field $\Delta\mathcal{E}(x, t)$ must be small compared to the potential electric field $E_0(t)$. It follows from formulae (20), (40), and (41) that the quantities $\mathcal{E}$ and $\Delta\mathcal{E}_m$, equation (44), are of the same order. Therefore, to ascertain the restrictions imposed by the condition $\mathcal{E} \ll E_0(t)$, we can use the quantity $\Delta\mathcal{E}_m$ in the latter condition. The quantity $\Delta\mathcal{E}_m$ should be much less than the minimum value of the function $E_0(t)$, i.e., the magnitude of potential field (34) for $\cos\omega t = 0$. The desired inequality reads

$$d^2 \frac{h_m l}{HR} \ll \delta_n^2(\omega) \qquad \delta_n^2(\omega) = \frac{c^2}{4\pi\sigma_0\omega} \tag{60}$$

where $\delta_n(\omega)$ stands for the characteristic penetration depth of the AC field into a metal under the condition of a normal skin effect. Secondly, the nonuniform component of the magnetic field inside the film must necessarily be much less than h_m. This stems from the assumption that the AC magnetic field $h(x, t)$ should be quasi-uniform ($h(x, t) \simeq h_m \cos\omega t$) over the film thickness. The maximum value of the nonuniform correction can be estimated from the first of Maxwell's equations (8) as $(4\pi\sigma_{tr}\,\Delta\mathcal{E}_m\,d/c) \sim h_m(d/\delta)^2$, where the effective penetration depth $\delta(\omega)$ is equal to $\delta_n(\omega)(R/l)^{1/2}$. As a result, we arrive at the following requirement which ensures that the quasi-uniform approximation is applicable:

$$d^2 \frac{l}{R} \ll \delta_n^2(\omega). \tag{61}$$

Comparing the restrictions imposed by inequalities (60) and (61), we see that condition (60) is stricter at large AC amplitudes, $h_m > H$, while for small values of h_m one should use inequality (61).

We expect the nonlinear effects discussed here to be observed under the same conditions as described in [13]. Those measurements were carried out on W and Cd specimens at helium temperatures in the presence of a DC magnetic field parallel to the sample surface. We suggest that such an experimental scheme should be modified by applying a low-frequency AC magnetic field instead. Let us estimate the relevant frequencies at which the predicted phenomena may be revealed. For a sample with a thickness of $d = 10^{-3}$ cm, width $D = 0.5$ cm, electron free path $l = 10^{-1}$ cm, concentration of electrons $N = 10^{23}$ cm^{-3}, Fermi momentum $p_F = 10^{-19}$ g cm s^{-1}, and for values of the current $I \sim 10$ A and magnetic fields $h_m \sim H \sim 10$ Oe, the phenomena revealed in this paper should be distinctly pronounced. From equations (60) and (61) it follows that $\omega < 10^5$ s^{-1} for this particular case. We give special attention to the fact that the wave amplitudes need not be large in order to observe the effects discussed. Values of the amplitude of the order of 10 Oe are quite accessible at low frequencies.

The unusual manifestation of the specific magnetodynamic mechanism of nonlinearity discussed in the present paper calls for further investigation. In the above analysis we have restricted ourselves to the case of an isotropic electron spectrum. It would be interesting to investigate the influence of the anisotropy of the Fermi surface on the results obtained in this paper. Besides this, we have used the framework of the quasiclassical theory. However, in sufficiently thin films the transverse motion of trapped electrons should be quantized. Since the contribution of these electrons to the nonlinear conductivity of the film is dominant, the size quantization can give rise to new effects in the nonlinear response. Finally, a question arises concerning the interaction of an AC signal with a direct current at high frequencies when inequalities (60) and (61) are not satisfied and the distribution of the AC magnetic field over the sample thickness becomes strongly inhomogeneous.

References

[1] Dolgopolov V T 1980 *Sov. Phys.– Usp.* **23** 134

[2] Makarov N M and Yampol'skii V A 1991 *Sov. J. Low Temp. Phys.* **17** 285

[3] Babkin G I and Dolgopolov V T 1976 *Solid State Commun.* **18** 713

[4] Makarov N M and Yampol'skii V Ł 1983 *Sov. Phys.–JETP* **58** 357

[5] Fisher L M, Voloshin I F, Makarov N M and Yampol'skii V A 1993 *J. Phys.: Condens. Matter* **5** 8741

[6] Kaner E A, Makarov N M, Yurkevich I V and Yampol'skii V C 1987 *Sov. Phys.–JETP* **66** 158

[7] Vugalter G A and Demikhovskii V Ya 1976 *Sov. Phys.–JETP* **43** 739

[8] Skobov V G and Chernov A S 1996 *Sov. Phys.–JETP* **82** 535

[9] Fisher L M, Makarov N M, Vekslerchik V E and Yampol'skii V A 1995 *J. Phys.: Condens. Matter* **7** 7549

[10] Makarov N M, Tkachev G B and Vekslerchik V E 1998 *J. Phys.: Condens. Matter* **10** 1033

[11] Kaner E A, Makarov N M, Snapiro I B and Yampol'skii V A 1984 *Sov. Phys.–JETP* **60** 1252

[12] Fuchs K 1938 *Proc. Camb. Phil. Soc.* **38** 100

[13] Voloshin I F, Kravchenko S V, Podlevskikh N A and Fisher L M 1985 *Sov. Phys.–JETP* **62** 132